

1. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is increasing or decreasing.
2. Find the intervals in which $f(x) = (x + 1)^3 (x - 3)^3$ is increasing or decreasing.
3. Find the maximum and minimum values of $y = \tan x - 2x$.
4. Find the maximum value of $\left(\frac{1}{x}\right)^x$.
5. A window in the form of rectangle surmounted by a semi-circular opening. The total perimeter of window is 10 m. Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening.
6. A closed cylinder has volume 2156 cm^3 . What will be the radius of its base so that its total surface area is minimum.
7. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius r is a square of side $\sqrt{2} r$.
8. Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm.
9. Find the semivertical angle of cone of maximum volume and given slant height.
10. Show that volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle θ is $\frac{4}{27} \pi h^3 \tan^2 \theta$.
11. Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2, -8)$.
12. A wire of 28 m is cut into two pieces. One of the pieces is to be made into a square and other into a circle. What should be the lengths of the two pieces so that the combined area of the circle and the square is minimum.
13. Find the values of k for which $f(x) = Kx^3 - 9Kx^2 + 9x + 3$ is increasing on \mathbb{R} .
14. Find the least value of 'a' such that the function $x^2 + ax + 1$ is increasing on $[1, 2]$.
15. If $f(x) = \sin^4 x + \cos^4 x$ where $x \in [0, \pi/2]$. Find the intervals in which the function $f(x)$ is increasing or decreasing.
16. Find the intervals in which the function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing or decreasing.
17. Prove that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is increasing on \mathbb{R} .
18. Find the intervals in which $f(x) = (x + 2)e^{-x}$ is increasing or decreasing.
19. If $y = a \log |x| + bx^2 + x$ has its extreme values at $x = -2$ and $x = -1$. Find the values of a and b .
20. Find the vertical angle of a right circular cone of minimum curved surface area that circumscribes a given sphere.