1. (i) What is the remainder when 5^{99} is divided by 13.

(ii) What is the coefficient of x^{53} in expansion $\sum_{m=0}^{100} {}^{100}C_m \, (x-3)^{100-m} \, 2^m$

(iii) Which of $99^{50} + 100^{50}$ and $(101)^{50}$ is large.

(iv) Prove that the value of the expression ${}^{47}C_4 + \sum_{J=1}^{5} {}^{52-J}C_3$ is ${}^{52}C_4$

(v) What is the sum of series : $\sum_{r=0}^{20} {}^{20}C_r$

2. (i) If the 6th term in the expansion of $\left(x^{\frac{1}{8/3}} + x^2 \log_{10} x \right)^8$ is 5600, what is x ?

(ii) How many integral terms are there in the expansion of $(5^{1/2} + 7^{1/8})^{1024}$

(iii) What is the coefficient of term independent of x in the expansion of $\left(1 + x + 2x^3\right)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

(iv) If the coefficients of rth and (r + 1)th terms in the expansion of $(3 + 7x)^{29}$ are equal. What is r.

(v) If third term in the expansion of $\left(\frac{1}{x} + x^{\log_{10} x}\right)^5$, $x > 1$ is 1000 , what is x.

3. (i) If $C_0, C_1, C_2 \ldots\ldots C_n$ denote the coefficients in the binomial expansion of $(1 + x)^n$, then prove that

(a) $C_0 + 3C_1 + 5C_2 + \ldots\ldots (2n + 1)\, C_n = (n + 1)\, 2^n$.

(b) $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots C_n^2 = \frac{(2n)!}{(n!)^2}$

(ii) If the sum of coefficients in the expansion of $(1 + 2x)^n$ is 6561. What is the coefficient of greatest term for x = 1/2.

(iii) Prove that the ratio of coefficient of x^{10} in $(1 - x^2)^{10}$ and the term independent of x in $\left(x - \frac{2}{x}\right)^{10}$ is 1 : 32.

(iv) If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1 + x)^n$, Prove that

$\frac{a_1 + a_2}{a_1}, \frac{a_2 + a_3}{a_2}, \frac{a_3 + a_4}{a_3}$ are in H.P.

(v) Using binomial theorem show that $2^{3n} - 7n - 1$ is divisible by 49. Hence show that $2^{3n+3} - 7n - 8$ is divisible by 49, $n \in N$.

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4. (i) If 2nd, 3rd and 4th, 5th and 6th terms in the expansion of $(x + a)^n$ are 240, 720 and 1080 respectively. Find x, a and n.

(ii) If 3rd, 4th, 5th and 6th terms in the expansion of $(x + y)^n$ be respectively a, b, c and d. Prove that $\frac{b^2 - ac}{c^2 - bd} = \frac{5a}{3C}$

(iii) If $C_0, C_1, C_2 \ldots. C_n$ denote the binomial coefficients in the expansion of $(1 + x)^n$, then prove that

$$C_0^2 - C_1^2 + C_2^2 - C_3^2 \ldots\ldots.(-1)^n C_n^2 = 0, \text{ If n is odd}$$
$$= (-1)^{n/2}\ {}^nC_{n/2}, \text{ If n is even}$$

(iv) If the fourth term in the expansion of $\left(\sqrt{x^{\frac{1}{\log x + 1}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200 and $x > 1$, then what is the value of x .

(v) If the coefficients of 5th, 6th and 7th terms in the expansion of $(1 + x)^n$ are in A.P. Find n.